Story of Maths - Part 2.2: Indian mathematics 19:45 - 38:00

B. Indian mathematics

Like the Chinese, the Indians had discovered the mathematical benefits of the decimal place-value system and were using it by the middle of the 3rd century AD.

Zero - Brahmagupta

We may never know how the Indians came up with their number system, creating the ancestors for the nine numerals used across the world now. But there was one number missing, and it was the Indians who would introduce it to the world. The earliest known recording of this number dates from the 9th century. This strange new numeral is engraved on the wall of small temple in the fort of Gwalior in central India. It's zero. The Indians transformed zero from a mere placeholder into a number that made sense in its own right - a number for calculation, for investigation. This brilliant conceptual leap would revolutionise mathematics. Perhaps there is also a cultural reason for the invention of zero. For the ancient Indians, the concepts of nothingness and eternity lay at the very heart of their belief system.

In the 7th century, the brilliant Indian mathematician Brahmagupta proved some of the essential properties of zero. Brahmagupta's rules about calculating with zero are taught in schools all over the world to this day. But Brahmagupta came a cropper when he tried to do one divided by zero. After all, what number times zero equals one?

Infinity - Bhaskara II

It would require a new mathematical concept, that of infinity, to make sense of dividing by zero, and the breakthrough was made by a 12th-century Indian mathematician called Bhaskara II, and it works like this: If I take a fruit and I divide it into halves, I get two pieces, so one divided by a half is two. If I divide it into thirds, I get three pieces. So when I divide it into smaller and smaller fractions, I get more and more pieces, so ultimately, when I divide by a piece which is of zero size, I'll have infinitely many pieces. So for Bhaskara, one divided by zero is infinity.

Negative numbers- Brahmagupta

But the Indians would go further in their calculations with zero. For example, if you take three from three and get zero, what happens when you take four from three? It looks like you have nothing, but the Indians recognised that this was a new sort of nothing - negative numbers. The Indians called them "debts".

The Indians' abstract approach to mathematics soon revealed a new side to the problem of how to solve quadratic equations. Brahmagupta's understanding of negative numbers allowed him to see that quadratic equations always have two solutions, one of which could be negative. Brahmagupta went even further, solving quadratic equations with two unknowns, a question which wouldn't be considered in the West until 1657, when French mathematician Fermat challenged his colleagues with the same problem. Brahmagupta was beginning to find abstract ways of solving equations, but astonishingly, he was also developing a new mathematical language to express that abstraction. He was experimenting with ways of writing his equations down, using the initials of the names of different colours to represent unknowns in his equations. A new mathematical language was coming to life, which would ultimately lead to the x's and y's we use today.

Trigonometry - Madhava

But Indian mathematicians also made fundamental new discoveries in the theory of trigonometry (which allows to translate geometry into numbers and back. At its heart lies the study of right-angled triangles. The sine function enables you to calculate distances when you're not able to make an accurate measurement. To this day, it's used in architecture and engineering. The Indians used it to survey the land around them, navigate the seas and, ultimately, chart the depths of space itself. The Indian astronomers could use trigonometry to work out the relative distance between Earth and the moon and Earth and the sun. The ancient Greeks had been the first to explore the sine function, listing precise values for some angles, but they couldn't calculate the sines of every angle. The breakthrough in the search for the sine function of every angle would be made in Kerala in south India.

In the 15th century, this part of the country became home to one of the most brilliant schools of mathematicians to have ever worked. Their leader was called Madhava. He discovered that you could add up infinitely many things with dramatic effects. Such infinite sums are called infinite series, and Madhava was doing a lot of research into the connections between these series and trigonometry. He realised that he could use the same principle of adding up infinitely many fractions to capture one of the most important numbers in mathematics - Pi. It was in 6th-century India that the mathematician Aryabhata gave a very accurate approximation for Pi - namely 3.1416. He went on to use this to make a measurement of the circumference of the Earth, and he got it as 24,835 miles, which amazingly is only 70 miles away from its true value.

But it was in Kerala in the 15th century that Madhava realised he could use infinity to get an exact formula for Pi. By successively adding and subtracting different fractions, Madhava could hone in on an exact formula for Pi, that is $\pi = 4\left[1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots\right]$. This formula is usually known as "Leibniz formula", and was thought to have been discovered by the 17th-century German mathematician Leibniz; but amazingly, it was actually discovered in Kerala two centuries earlier by Madhava. He went on to use the same sort of mathematics to get infinite-series expressions for the sine formula in trigonometry.