APPROXIMATING π WITH MADHAVA'S METHOD - SOLUTION

Madhava's formula : $\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} ...\right]$.

1) What is the first definition of the number Pi, and why is it called π ?

$\pi = \frac{\text{a circle's circumference}}{\text{this circle's diameter}}$, and the letter π (p in ancient Greek), is the first letter of "perimeter".

2) Why can't we simply write the number Pi with a decimal point?

Because Pi isn't a decimal number; it is irrational!

3) Why do you think this formula is called an "infinite series"?

Because it's a sum with an infinite number of terms.

4) Using Madhava's formula, calculate an approximation of Pi:
- with a sum of 3 terms inside the square brackets : 3.46666666667
- with a sum of 5 terms inside the square brackets : 3.33968253968
- with a sum of 7 terms inside the square brackets : 3.28373848374

What do you notice? It seems to be converging towards Pi.

Gregory-Leibniz	3.21840276593	3.178617011	3.16597927285	3.15977296977	3.1560846464
Series:	3.07025461778	3.10588973827	3.11778650176	3.12373693373	3.12730766798
4.0	3.20818565226	3.17606517687	3.16484532529	3.15913516382	3.15567646231
2.66666666667	3.0791533942	3.1082685667	3.1188683138	3.12435255512	3.12770443434
3.46666666667	3.20036551541	3.17384233719	3.16381213402	3.15854058931	3.15529064124
2.89523809524	3.08607980113	3.1103502737	3.11985609006	3.12492714393	3.12807975688
3.33968253968	3.19418790924	3.17188873524	3.16286684275	3.15798499517	3.15492539447
2.97604617605	3.09162380667	3.1121872427	3.1207615796	3.12546466997	3.12843532824
3.28373848374	3.18918478228	3.1701582572	3.161998693	3.15746466997	3.15457911909
3.01707181707	3.09616152647	3.11382022903	3.1215946526	3.12596860698	3.12877266748
3.25236593472	3.18505041536	3.16861474958	3.16119861299	3.15697635891	3.15425037449
3.04183961893	3.09994403238	3.11528141624	3.12236366154	3.12644200777	
3.23231580941	3.18157668544	3.16722946819	3.16045889963	3.15651719574	... etc ...
3.05840276593	3.10314531289	3.1165965568	3.12307572206	3.12688756611	

5) Using the fact that you can write any odd number under the form $2n+1$, and that the alternating signs can be written $(-1)^n$, write down Madhava's formula using the sign $\sum_{n \geq 0}$ (which means "sum for all integers $n \geq 0$").

$$\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} ...\right] = 4 \times \sum_{n \geq 0} (-1)^n \times \frac{1}{2n+1}$$

6) Another infinite series which gives an approximation for Pi is the Nilakantha method. Kelallur Nilakantha Somayaji (1444–1544) was also a major mathematician and astronomer of the Kerala school of astronomy and mathematics.
His formula is:

$$\pi = 3 + \frac{4}{2 \times 3 \times 4} - \frac{4}{4 \times 5 \times 6} + \frac{4}{6 \times 7 \times 8} - \frac{4}{8 \times 9 \times 10} + ...$$

Using Nilakantha's formula, calculate an approximation of Pi:
- with a sum of 3 terms : 3.13333333
- with a sum of 5 terms : 3.13968254
- with a sum of 7 terms : 3.14088134

What do you notice? Can you compare the two formulas?

Nilakantha's formula seems to converge towards Pi faster than Madhava's formula.