1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
→ 1 6 15 20 15 6 1

• z_1 solution de $z^6 + 8 = 0$ ssi $z_1^6 = -8$.

$$z_1^6 = \left[\sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right]^6 = \sqrt{2}^6 \left(\frac{\sqrt{3} - i}{2}\right)^6 = 2^3 \times \frac{(\sqrt{3}-i)^6}{2^6}$$

$$= \frac{1}{2^3}\left[\sqrt{3}^6 + 6\sqrt{3}^5 \times (-i) + 15\sqrt{3}^4 \times (-i)^2 + 20\sqrt{3}^3 (-i)^3 + 15\sqrt{3}^2 \cdot (-i)^4 + 6\sqrt{3} \cdot (-i)^5 + (-i)^6\right]$$

$$= \frac{1}{2^3}\left[3^3 + 6 \times 3^2\sqrt{3} \times (-i) + 15 \times 3^2 \times (-i)^2 + 20 \times 3\sqrt{3} \times (-i)^3 + 15 \times 3 \times (-i)^4 + 6\sqrt{3} \cdot (-i)^5 + (-i)^6\right]$$

$$= \frac{1}{2^3}\left[27 - 54\sqrt{3}i + 135 \times (-1) + 60\sqrt{3} \times (-i) + 45 \times 1 - 6\sqrt{3} \times i + (-1)\right]$$

$$= \frac{1}{2^3}\left[27 - 54\sqrt{3}i - 135 + 60\sqrt{3}i + 45 - 6\sqrt{3}i - 1\right]$$

$$= \frac{1}{2^3} \times [-64] = -8$$

• $z_6 = \overline{z_1}$, donc $z_6^6 = \overline{z_1}^6 = \overline{z_1^6} = \overline{-8} = -8$,

car le conjugué d'un réel est ce réel lui-même.

• z_2 est solution ssi $z_2^6 = -8$

$$z_2^6 = \left[\sqrt{2} \times (-i)\right]^6 = \sqrt{2}^6 \times (-i)^6 = 2^3 \times \left((-i)^2\right)^3$$

$$= 8 \times (-1)^3 = -8$$

- $z_5 = \overline{z_2}$, donc $z_5^6 = \overline{z_2}^6 = \overline{z_2^6} = \overline{-8} = \underline{-8}$

- z_3 est solution ssi $z_3^6 = -8$.

$z_3^6 = \left[\sqrt{2}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right]^6 = \sqrt{2}^6 \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^6$

$= 2^3 \left[\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \times (-1)\right]^6 = 2^3 \left[\frac{\sqrt{3}}{2} + \frac{1}{2}i\right]^6 \times \underbrace{(-1)^6}_{=1}$

$= 2^3 \times \left(\frac{\sqrt{3}+i}{2}\right)^6 = \frac{2^3}{2^6} \times (\sqrt{3}+i)^6$

$= \frac{1}{2^3}\left(\sqrt{3}^6 + 6\sqrt{3}^5 i + 15\sqrt{3}^4 i^2 + 20\sqrt{3}^3 i^3 + 15\sqrt{3}^2 i^4 \right.$

$\left. + 6\sqrt{3}i^5 + i^6\right)$

$= \frac{1}{2^3}\left(3^3 + 6 \times 3^2\sqrt{3}i + 15 \times 3^2 \times i^2 + 20 \times 3\sqrt{3}i^3 + 15 \times 3 i^4 \right.$

$\left. + 6\sqrt{3}i^5 + i^6\right)$

$= \frac{1}{2^3}\left(27 + 54\sqrt{3}i + 135 i^2 + 60\sqrt{3}i^3 + 45 i^4 + 6\sqrt{3}i^5 + i^6\right)$

$= \frac{1}{2^3}\left(27 + 54\sqrt{3}i - 135 - 60\sqrt{3}i + 45 + 6\sqrt{3}i - 1\right)$

$= \frac{1}{2^3} \times [-64] = \underline{-8}$

(Dans ce dernier cas, il est beaucoup plus rapide de remarquer que $z_3 = -z_6$, donc $z_3^6 = (-z_6)^6 = z_6^6 = \underline{-8}$

- $z_4 = \overline{z_3}$, donc $z_4^6 = (\overline{z_3})^6 = \overline{z_3^6} = \overline{-8} = \underline{-8}$